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AESB2341
Rock Mechanics and Rock Engineering
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CT-IZ 1.95 / 1.98

Exam : answer model

Question 1 (30 points)

An unconfined compression test was performed on a cylindrical sample of granite. The initial height of the sample is equal to $l_0 = 110$ mm and the initial radius is equal to $r_0 = 25$ mm. The changes in height Δl and in radius Δr are recorded as a function of the applied axial force F (Table 1). Δl is taken positive for a decrease in height, while Δr is positive for an increase in radius.

F (kN)	Δl (mm)	Δr (mm)	**Axial stress** (MPa)	**Axial strain** (-)	**Radial strain** (-)
0.0	0.000	0.000	0.0	0.000	0.000
170.0	0.185	0.011	86.58	$1.68 \cdot 10^{-3}$	$4.40 \cdot 10^{-4}$
370.0	0.400	0.024	188.44	$3.64 \cdot 10^{-3}$	$9.60 \cdot 10^{-4}$
471.5	0.480	0.028	240.13	$4.36 \cdot 10^{-3}$	$1.12 \cdot 10^{-3}$
589.2	0.590	0.041	300.08	$5.36 \cdot 10^{-3}$	$1.64 \cdot 10^{-3}$
0.1	0.620	0.045	0.05	$5.64 \cdot 10^{-3}$	$1.80 \cdot 10^{-3}$

Table 1 – Experimental results from an unconfined compression test on granite.

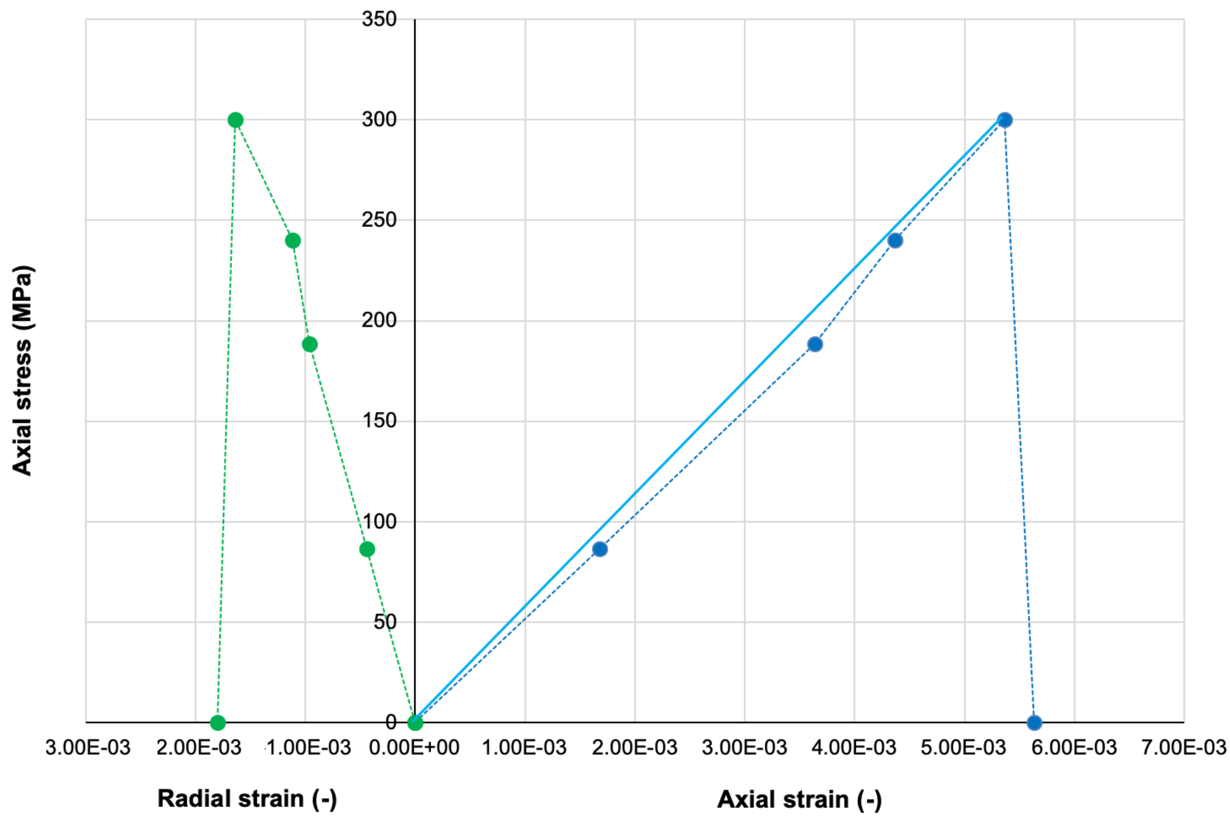
a The experimental results from mechanical tests are generally represented under the form of stress – strain curves. Give the definitions and units of the axial stress, axial strain and radial strain. [**6 points**]

- Axial stress : $\sigma_a = F/A$, where F is the axial force and A is the cross-sectional area. It has units of N/m^2.
- Axial strain : $\varepsilon_a = \Delta l/l_0$, where Δl is the change in sample height and l_0 is the initial height. It has no unit or is expressed in %.
- Radial strain : $\varepsilon_r = \Delta r/r_0$, where Δr is the change in sample radius and r_0 is the initial radius. It has no unit or is expressed in %.

b Explain why stresses and strains are used instead of forces and changes in dimensions. [**3 points**]

For a given material, the relationship between the applied forces and the resulting changes in dimensions (or *vice versa*) is not unique, but depends on the sample size and geometry. Stresses and strains are used instead in order to bypass this limitation.

c Fill in Table 1 by calculating the axial stress, axial strain and radial strain. Draw below the axial stress – axial strain curve and the axial stress – radial strain curve of the tested specimen. [**5 points**]



d Determine the secant Young's modulus and the Poisson's ratio of the tested granite. Refer to the stress – strain curves drawn above to explain your calculations. [**4 points**]

The secant Young's modulus is given by the slope of the line joining (0,0) to the peak of the axial strain – axial stress curve :

$$E_{sec} = \frac{300.08}{5.36 \cdot 10^{-3}} = 55947 \text{ MPa}$$

The Poisson's ratio is given by the ratio between the radial strain and the axial strain, in the elastic region :

$$\nu = \frac{1.12 \cdot 10^{-3}}{4.36 \cdot 10^{-3}} = 0.26$$

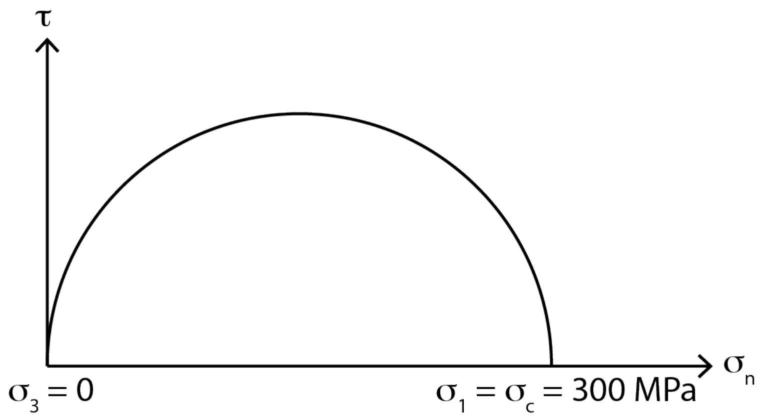
e Determine the unconfined compressive strength of the tested granite. [**2 points**]

The unconfined compressive strength is given by the peak of the stress - strain curve :

$$\sigma_c = 300.08 \text{ MPa}$$

f Draw the Mohr circle at failure and explain what the Mohr circle represents. [**4 points**]

The Mohr circle is a graphical representation of the stress tensor and represents the state of stress at a point. The Mohr circle of the sample at failure ($\sigma_3 = 0$, $\sigma_1 = \sigma_c$) is given by :



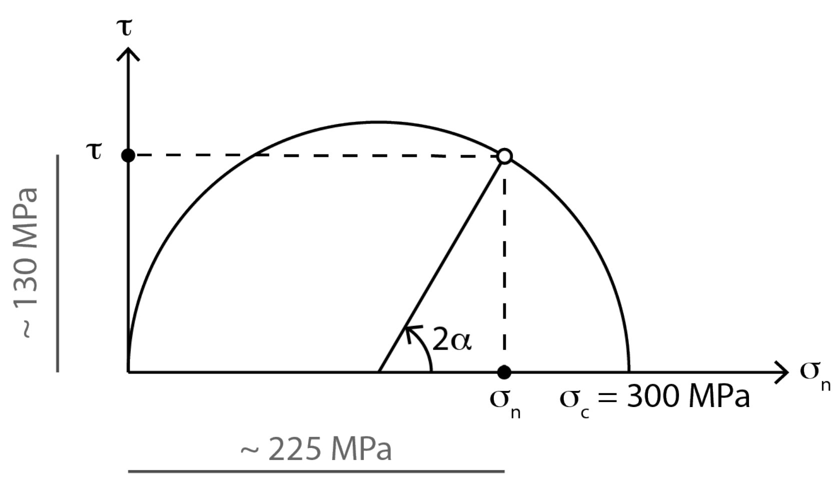
g Determine, analytically and graphically on the Mohr circle, the state of stress at failure on a plane inclined of an angle of 30° to the horizontal. [**6 points**]

The state of stress (normal stress σ_n and shear stress τ) at failure on a plane inclined of an angle of $\alpha = 30°$ to the horizontal can be determined analytically using the transformation equations :

$$\sigma_n = \sigma_3 \sin^2 \alpha + \sigma_1 \cos^2 \alpha = \sigma_c \cos^2 \alpha = 225.06 \text{ MPa}$$

$$\tau = -(\sigma_3 - \sigma_1) \sin \alpha \cos \alpha = \sigma_c \sin \alpha \cos \alpha = 129.94 \text{ MPa}$$

and graphically on the Mohr circle, from which we get $\sigma_n \simeq 225$ MPa and $\tau \simeq 130$ MPa :



Question 2 (15 points)

The *in situ* state of stress in a granitic rock mass was measured at a depth of 500 m by the hydraulic fracturing technique. Assume the ground is saturated continuously from the surface and that the water pressure in the ground is hydrostatic. The water pressure was first raised 3.5 MPa above the original groundwater pressure and then it was not possible to raise it further. When pumping was stopped, the water pressure fell to a value 0.5 MPa above original groundwater pressure. Two days later, the pressure was raised again, but it could not be pumped to a value higher than 1.0 MPa above the previous pressure.

a Calculate the breakdown pressure P_f and the shut-in pressure P_s. Use $\rho_w = 1000$ kg/m^3 and $g = 9.8$ m/s^2. [**3 points**]

The water pressure at a depth of 500 m is equal to :

$$p_w = \rho_w \cdot g \cdot 500 = 4.9 \text{ MPa}$$

The breakdown pressure is equal to :

$$P_f = 3.5 + 4.9 = 8.4 \text{ MPa}$$

The shut-in pressure is equal to :

$$P_s = 4.9 + 0.5 = 5.4 \text{ MPa}$$

b Give the mathematical expressions and calculate the minor and major horizontal stresses. [**5 points**]

The minor horizontal stress is given by :

$$\sigma_h = P_s = 5.4 \text{ MPa}$$

The major horizontal stress is given by :

$$\sigma_H = 3\sigma_h - P_r = 3 \cdot 5.4 - 6.4 = 9.8 \text{ MPa}$$

with :

$$P_r = P_s + 1.0 = 6.4 \text{ MPa}$$

c Calculate the tensile strength of the rock. [**2 points**]

$$T_0 = P_f - P_r = 8.4 - 6.4 = 2.0 \text{ MPa}$$

d How many hydraulic fracturing set-ups are required to determine the 3D state of stress in a rock mass? Explain your answer. [**3 points**]

Conventional hydraulic fracturing in a vertical borehole measures two values, the principal stresses are assumed to be parallel and perpendicular to the borehole axis, and the vertical stress value is assumed on the basis of gravitational stress. As a result, with two measured values and the equivalent of four assumptions, only one set-up is required to establish the stress tensor components.

e The *in situ* state of stress needs to be determined for a project of deep geothermal energy. Explain the advantage of the hydraulic fracturing technique over the flat jack technique. [**2 points**]

The flat jack technique requires direct access to a rock wall and is therefore not suited to determine the *in situ* state of stress at hight depths. This is not the case for the hydraulic fracturing technique.

Question 3 (15 points)

Trona is a non-marine evaporite mineral, which is mined as the primary source of sodium carbonate in the United States. It is found as a sub-horizontal stratiform orebody in the Green River Formation in Wyoming. The Green River Formation is found at a depth of 450 m and has a thickness of 4 m. The average density of the overlying shale is equal to 2110 kg/m^3.

The lateral extension of the Green River Formation makes it particularly suitable for room-and-pillar mining. The foreseen design considers 5.0-m rooms ($w_0 = 5.0$ m) and 7.0-m square pillars ($w_p = 7.0$ m), and extraction of the full stratigraphic thickness of 4 m.

Analysis of pillar failures in a nearby mine indicates that the pillar strength is defined by :

$$S \text{ (in MPa)} = 56.3 \cdot h^{-0.83} \cdot w_p^{0.5}$$

where h is the pillar height (in m) and w_p is the pillar width (in m).

a Calculate the pre-mining vertical stress. Use $g = 9.8$ m/s^2. [**2 points**]

$$\sigma_v = 450 \cdot \rho_r \cdot g = 450 \cdot 2110 \cdot 9.8 = 9.3 \text{ MPa}$$

b Calculate the stress in a pillar using the tributary area method. Explain your calculations. [**4 points**]

According to the tributary area method, each pillar supports the weight of a $(w_p + w_o)$-wide column of overburden. Accordingly, the average stress in a pillar is given by :

$$\sigma_p = \sigma_v \frac{(w_o + w_p)^2}{w_p^2} = 27.3 \text{ MPa}$$

c Calculate the strength of a pillar. [**1 point**]

$$S = 56.3 \cdot 4^{-0.83} \cdot 7^{0.5} = 47.1 \text{ MPa}$$

d Explain why the strength of pillars depends on their size. [**2 points**]

The pillar contains (micro-)cracks : the larger the pillar, the greater the number of (micro-)cracks and hence the greater the likelihood of a more severe flaw.

e Calculate the factor of safety against compressive failure of the pillars. [**1 point**]

$$FoS = \frac{\sigma_p}{S} = \frac{47.1}{27.3} = 1.73$$

f Define, give the mathematical expression and calculate the corresponding extraction ratio. [**3 points**]
The extraction ratio is the ratio between the volume of ore mined and the total volume of ore. Its mathematical expression is given by :

$$e = \frac{(w_0 + w_p)^2 - w_p^2}{(w_0 + w_p)^2} = 0.66$$

g Which failure mechanism other than compressive failure of pillars can happen in a room and pillar mine ? [**2 points**]

Bearing capacity failure of the roof or floor.

Question 4 (40 points)

A 12.0-m high rock slope has been excavated at a face angle of 60°. The rock in which this cut has been made contains persistent bedding planes that dip at an angle of 35° into the excavation (slide plane). A 4.35-m deep tension crack is 4.0 m behind the crest, and is filled with water to a height of 3.0 m above the slide plane (Figure 1). It is assumed that water escapes at atmospheric pressure where the slide plane daylights in the slope face.

The strength parameters of the slide plane are as follows :

— Cohesion, $c = 25$ kPa

— Friction angle, $\phi = 37°$

The unit weight of the rock is 26 kN/m^3, and the unit weight of the water is 9.8 kN/m^3.

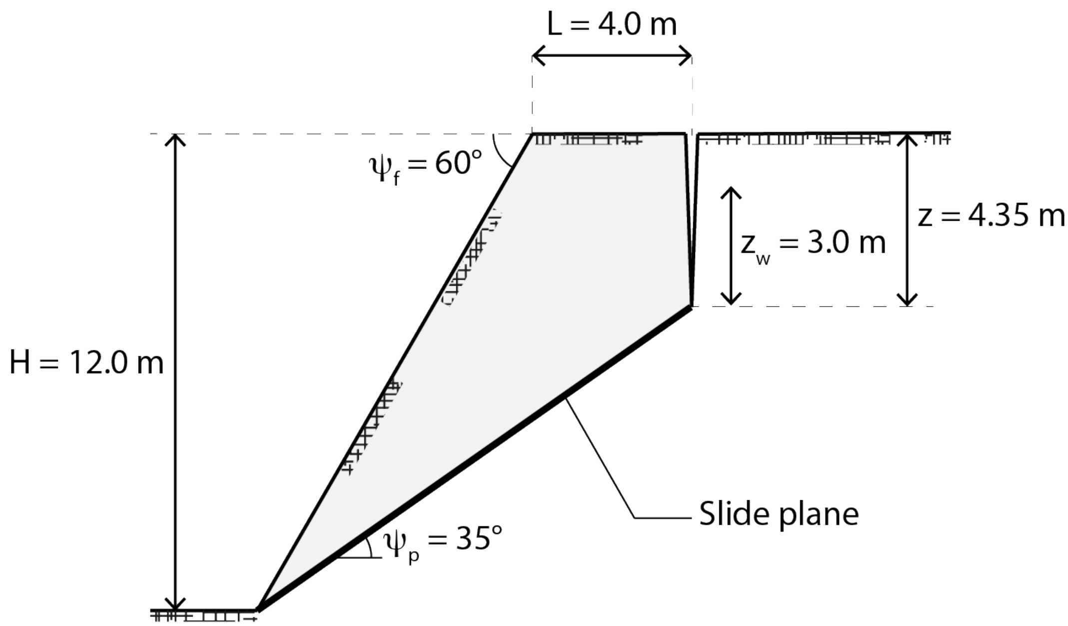


FIGURE 1 – Slope geometry.

a List two kinematic conditions for a plane failure to occur. [**3 points**]

- The potential sliding plane dips out of the slope.

- The dip of the slope must exceed that of the potential slide plane.

b List, label, represent and give the mathematical expression of the forces acting on the block shaded in Figure 1 at limit equilibrium conditions. [**10 points**]

Note : a 1-m thick slice of the slope is considered.

- W : weight of the rock block

$$W = \gamma_r \left[\left(L + \frac{H - z}{\tan \psi_p} \right) \frac{H}{2} - \frac{1}{2} \frac{(H - z)^2}{\tan \psi_p} \right]$$

- U : resultant force of water pressure on the sliding surface

$$U = \frac{1}{2} \gamma_w z_w \frac{H - z}{\sin \psi_p}$$

- V : resultant force of water pressure in the tension crack

$$V = \frac{1}{2} \gamma_w z_w^2$$

- R_ϕ : resisting force due to friction along the slide plane

$$R_\phi = (W \cos \psi_p - U - V \sin \psi_p) \tan \phi$$

- R_c : resisting force due to cohesion along the slide plane

$$R_c = c \frac{H - z}{\sin \psi_p}$$

c Develop a mathematical expression of the resultant force resisting failure along the slide plane. [**2 points**]

$$F_{resisting} = R_\phi + R_c$$

d Develop a mathematical expression of the resultant force driving failure along the slide plane. [**2 points**]

$$F_{driving} = W \sin \psi_p + V \cos \psi_p$$

e Give the mathematical expression and calculate the factor of safety against plane failure along the slide plane. [**5 points**]

$$FoS = \frac{F_{resisting}}{F_{driving}} = \frac{(W \cos \psi_p - U - V \sin \psi_p) \tan \phi + c \frac{H - z}{\sin \psi_p}}{W \sin \psi_p + V \cos \psi_p} = 1.25$$

f Is the rock mass stable or not ? Justify your answer. [**2 points**]

Yes, the factor of safety is larger than 1.

g Give the mathematical expression and calculate the factor of safety against plane failure if the cohesion were to be reduced to zero due to excessive vibrations from nearby blasting operations. [**2 points**]

$$FoS = \frac{F_{resisting}}{F_{driving}} = \frac{(W\cos\psi_p - U - V\sin\psi_p)\tan\phi}{W\sin\psi_p + V\cos\psi_p} = 0.80$$

h It is proposed the slope with zero cohesion be reinforced by installing tensioned rock bolts anchored into sound rock beneath the slide plane (Figure 2). If the rock bolts are installed at right angles to the slide plane, that is, $\psi_T = 55°$, and the total load on the anchors per lineal meter of slope (i.e. per 1-m thick slice of slope) is 400 kN, give the mathematical expression and calculate the factor of safety against plane failure. [**3 points**]

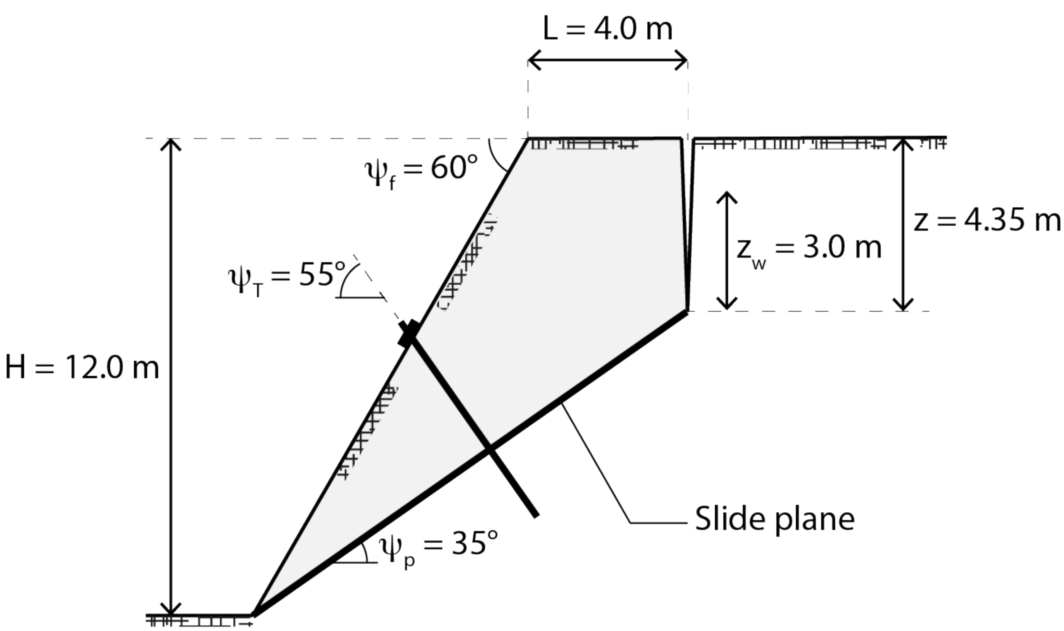


FIGURE 2 – Reinforced slope geometry.

Since the anchor is perpendicular to the slide plane, the factor of safety is given by :

$$FoS = \frac{F_{resisting}}{F_{driving}} = \frac{(W\cos\psi_p - U - V\sin\psi_p + T)\tan\phi}{W\sin\psi_p + V\cos\psi_p}$$

with $T = 400$ kN, so that :

$$FoS = 1.20$$

i Explain how rock bolts installed at right angles to the slide plane increase the factor of safety of the slope against failure along the slide plane. [**3 points**]

The tensioned rock bolts increase the normal force to the slide plane, thereby increasing the resisting force due to friction and the factor of safety.

j Based on the case described above, explain the difference between a risk and a hazard. [**3 points**]

A hazard is anything that can cause harm (possibility). For instance, slope failure represents a hazard. A risk is the chance of harm being done (probability). For instance, the risk of slope failure is reduced by stabilizing the slope with rock bolts.

k The limit-equilibrium analysis assumes that the strength of a discontinuity is described using the Mohr-Coulomb failure criterion. Comment on the validity of this hypothesis. [**5 points**]

The Mohr-Coulomb criterion is expressed as a linear relationship between the normal stress σ_n and the shear stress τ on a plane at failure :

$$\tau = c + \sigma_n \tan\phi$$

where c is the cohesion and ϕ is the friction angle.

However, in practice, the shear strength of discontinuity is a non-linear function of the applied normal stress over. In order to use the simple Mohr-Coulomb criterion, it is therefore important to determine and use shear strength parameters which are representative of the states of stress along the discontinuity is the rock slope.

Furthermore, the discontinuity properties can evolve with time, which is not taken into account by the Mohr-Coulomb failure criterion.